Significance of Group Delay Functions in Spectrum Estimation

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***Abstract*—In this paper we propose a method of spectrum estimation using group delay functions. This method exploits the additive property of the Fourier transform (FT) phase to extract spectral information of the signal in the presence of noise. The phase is generally featureless due to random polarity and wrapping; but the group delay function can be processed to derive significant information such as peaks in the spectral envelope. In the spectral estimates obtained using this method, the resolution properties of the periodogram estimate are preserved while the variance is reduced. Variance caused by the sidelobe leakage due to windows and additive noise are significantly reduced even in the spectral estimate obtained using a single realization of the observation peak. Resolution is primarily dictated by the size of the data window according to the standard time bandwidth product relation. The method works even for high noise levels (SNR = 0 dB or less). The results of this spectrum estimation procedure are demonstrated through several illustrative examples. In particular, two cases are considered, namely, 1) estimation of sinusoids in noise and 2) estimation of the narrow-band autoregressive process in noise.**

I. Introduction

The objective of this study is to explore an approach to spectrum estimation from the Fourier transform (FT) phase of a signal, rather than conventional spectrum estimation from the FT magnitude. The method described is based on the properties of the negative derivative of the FT phase function, also called group delay function. The most important properties of the group delay function are the additive and high resolution properties [1]. Here *resolution* refers to the sharpness of the peaks in the group delay function, which is due to the squared FT magnitude function behavior of the group delay functions near the peaks. The term *resolution* is used in a slightly different sense than what is normally used for the frequency resolving capability of a given spectrum estimation method. The key idea behind the new spectrum estimation method is that the properties of the group delay functions for noise and an autoregressive process are distinct. The main property of noise used in this study is that the noise samples are uncorrelated. The proposed processing method does not use the long-term statistical properties of noise. It is based on reducing the effect of the uncorrelated noise samples to enhance the correlated signal component in the group delay function. The proposed spectrum estimation method is applicable for both sinusoids in noise as well as for narrow-band autoregressive processes.

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Spectrum analysis of signals is performed to extract information about the system that generated the signal. Since the signal available for analysis is usually of short duration and also noisy, one can only attempt to estimate a) the spectrum or b) the system characteristics, rather than compute the true spectrum. The accuracy of the estimated spectrum depends on the bias and variance of the estimate, which in turn depends on the nature of the signal, its duration, type of windowing, and noise. In most studies on spectrum estimation two classes of problems are addressed [2], [3]: 1) estimation of AR parameters or AR spectrum from finite data, and 2) estimation of component sinusoids from finite duration noisy data.

The classical approaches to spectrum estimation are based on statistical properties of signals and noise. These approaches have a severe limitation in that they require large data records. Methods based on the periodogram are examples of such an approach. Many modern methods involving models and parameter estimation have been proposed in the literature [2], [3] to deal with short data records. Performance of the model-based methods depend critically on the accuracy of representation of the random process by the model.

The periodogram method has many advantages, such as high resolution and small bias in the estimated spectrum. In fact, even for high noise levels (SNR < -10 dB) one can detect the presence of sinusoids in the computed spectrum. The main difficulty with periodogram is that the variance of the estimate is as large as the actual spectral value itself. Reduction of the variance by averaging either increases the bias of the estimate or requires that many realizations of the same process are available. The variance caused by additive noise cannot be reduced by increasing the length of the data. Variance is also caused by the sidelobes of the data windows. Windows with small sidelobe leakage have less resolution. The issues in spectrum estimation are therefore [3] a) conflict between signal detectability and spectrum estimation, (b) resolution from limited size of data record, and (c) choice between parameter estimation and spectral estimation. In this study an attempt is made to retain the advantages of periodogram without compromising on the resolution.

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We explore a method in which the variance of the estimated power spectrum is reduced. No attempt is made to obtain higher resolution than is available in the periodogram. Therefore the problem of short data records is not addressed in this paper.

In all the studies on spectrum estimation so far, the emphasis was only on the FT magnitude (since the power spectrum is square of the FT magnitude). Our research objective is to show that it is possible to extract information about the embedded sinusoids or autoregressive process in noise by using the FT phase also. In Section II, the characteristics of FT phase and the negative derivative of the phase, i.e., the group delay are discussed. The relationship between the FT phase and magnitude of a signal through group delay functions is also established in Section II. A spectrum estimation method based on processing the group delay function is discussed in Section III. In Section IV the effectiveness of the proposed method for estimation of sinusoidal components and autoregressive process from signals corrupted with noise is illustrated through examples.

II. Characteristics of FT Phase and Group Delay Functions

A. Group Delay Functions: Relation Between FT Magnitude and FT Phase

Given a discrete-time real signal $x(n)$, the z transform is given by

$$X(z) = \sum_n x(n) z^{-n}. \quad (1)$$

We can write $X(z)$ as

$$X(z) = \prod_i X_i(z) \quad (2)$$

where $X_i(z)$ is either a first-order or a second-order polynomial with real coefficients. The roots of $X_i(z)$ are either real or a complex conjugate pair. The Fourier transform $X(\omega)$ of $x(n)$ is obtained by evaluating $X(z)$ on the unit circle in the z plane. The FT magnitude is a product of the magnitudes of the individual components, and the FT phase is a sum of the phases of the individual components.

The relation between the FT magnitude and the FT phase can be seen clearly through the cepstral domain [4]. To establish the relation between the magnitude $|X(\omega)|$ and the phase $\theta(\omega)$ of the FT of a signal, we can define two group delay functions, $\tau_m(\omega)$ and $\tau_p(\omega)$ corresponding to $\log |X(\omega)|$ and $\theta(\omega)$, respectively, as follows: Let

$$\log |X(\omega)| = \sum_n c_1(n) \cos \omega n \quad (3)$$

$$\theta(\omega) = -\sum_n c_2(n) \sin \omega n \quad (4)$$

$$\tau_p(\omega) = -d\theta(\omega)/d\omega = \sum_n n c_2(n) \cos \omega n \quad (5)$$

and

$$\tau_m(\omega) = \sum_n n c_1(n) \sin \omega n. \quad (6)$$

Then we can show that

$$\tau_p(\omega) = \sum_i \tau_{p_i}(\omega) \quad (7)$$

and

$$\tau_m(\omega) = \sum_i \tau_{m_i}(\omega) \quad (8)$$

where $\tau_{p_i}(\omega)$ and $\tau_{m_i}(\omega)$ are the group delay functions for the component polynomials.

$\tau_p(\omega)$ is normally referred to as the group delay function. For simple first-order or second-order polynomials the shapes of $\tau_p(\omega)$ are shown in Fig. 1. Properties of group delay functions follow from the discussion of the group delay functions of minimum and maximum phase signals as described in [5]. In particular, if all the roots of $X(z)$ lie inside the unit circle in the z plane, then

$$\tau_p(\omega) = \tau_m(\omega). \quad (9)$$

On the other hand, if all the roots of $X(z)$ lie outside the unit circle in the z plane, then

$$\tau_p(\omega) = -\tau_m(\omega). \quad (10)$$

If the roots of $X(z)$ are distributed both inside and outside the unit circle, then the component group delay functions follow either the relation (9) or the relation (10) depending on whether the root, is inside or outside the unit circle, respectively.

B. Distribution of Roots of $X(z)$ for Noise Data

The autocorrelation function of a noise sequence is an impulse at zero lag and zero at other lags. Such a noise sequence has its z-transform roots distributed randomly close to the unit circle in the z plane. The roots may be both inside and outside the unit circle. Fig. 2(a) shows typically the distribution of zeros in the z plane for a noise sequence that is 40 samples in length and Fig. 2(b) shows the corresponding group delay function $\tau_p(\omega)$. The group delay function for a noise sequence will have large spikes (both positive and negative) at random locations along the frequency axis. The values of the group delay function at frequencies other than the root frequencies are very small.

The behavior of $\tau_p(\omega)$ at the root frequencies can be understood from the results on power spectrum of noise using the periodogram method [2]. In particular, the expressions for bias and variance of the power spectrum estimates can be used to study the bias and variance properties of the group delay function at the root frequencies. First, it is interesting to see the shapes of $\tau_p(\omega)$ near the root frequencies for a first-order and a second-order polynomial. The expression for $\tau_p(\omega)$ is given by [1]

$$\tau_p(\omega) \approx C/P(\omega), \qquad \text{for } \omega \text{ near the root frequency} \quad (11)$$

where C is a constant, and $P(\omega)$ is the power spectrum. For a signal contributed by an all-zero or an all-pole or a

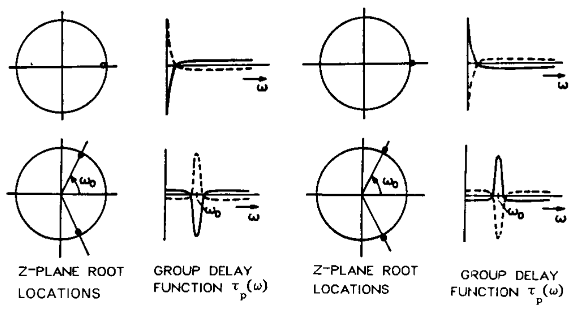
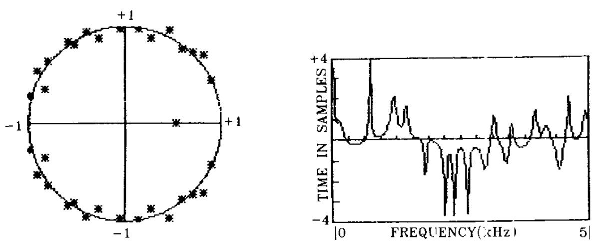


Fig. 1. Illustration of the group delay functions for different first- and second-order polynomials. The dotted curves correspond to poles in the z plane at the same locations as the zeros.



(a) (b)

Fig. 2. Behavior of zeros for a noise sequence of length 40 samples. (a) Distribution of zeros in the z plane and (b) the corresponding group delay function.

pole-zero filter, the shape of the group delay function near a root frequency due to individual components (roots of the polynomials, poles or zeros) is inversely proportional to the power spectrum value contributed by the root. The constant of proportionality will have appropriate sign depending on whether the root is inside or outside the unit circle, and whether the root is a pole or a zero. From the studies [2], [3] on bias and variance of periodograms estimates of power spectrum of noise sequences, we know that as the number of samples are increased the estimated spectrum is unbiased, but the variance becomes very large, almost equal to the value of the spectrum itself. This implies that the spikes in the group delay function at the root frequencies are unreliable.

C. Properties of the Group Delay Function of an All-Pole Model

The group delay function of a stable all-pole system $(1/A(z))$ corresponding to an autoregressive process is contributed by the component polynomials $A_i(z)$ of the system. Since all the roots are away from the unit circle, the contributions of the individual components are broader compared to the spikes due to noise components. A typical plot of the group delay function for an all-pole system is shown in Fig. 3(a). Note that here also the significant values are concentrated around the root frequencies. Fig. 3(b) shows the group delay function plot for a noise sequence. Note the different scales in Figs. 3(a) and (b).

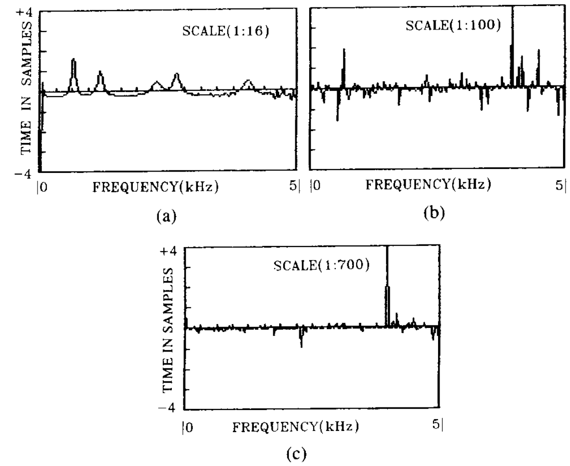


Fig. 3. Group delay functions for different signals. (a) Group delay function corresponding to the impulse response of a stable all-pole system. (b) Group delay function corresponding to a random noise sequence. (c) Group delay function corresponding to the response of an all-pole system to random noise (note the different vertical scales in each plot).

The combined response shown in Fig. 3(c) is for the synthetic data generated by convolving the all-pole system response with the noise sequence. Fig. 3(c) highlights the main difficulty in processing the group delay function. The characteristics of the all-pole system are completely masked by the dominant spikes due to noise. Moreover, the finite duration of the signal and the window effects will not result in the strict addition of the component group delay functions. But fortunately the distinct characteristics of the group delay functions of noise and the all-pole system enable us to separate their effects. In fact, we can reduce the effect of spikes in Fig. 3(a) to bring out the features of the system. This forms the basis for the spectrum estimation procedure to be described in Section III. Since the method works well even for additive noise, as will be shown later with illustrations, it can be adopted for the general problem of spectrum estimation.

III. Spectral Estimation from Group Delay Function

As mentioned before, the objective of this paper is to estimate the spectral features of an autoregressive process or a sinusoidal process in noise using the properties of Fourier transform phase, or equivalently using group delay functions. Methods proposed earlier [6]–[8] processed the FT phase or group delay function indirectly to extract useful spectral information. In this section a method of spectrum estimation based on processing the group delay function directly is proposed. Application of this method for speech processing is described in [9], [10].

Let us consider the output $x(n)$ of an autoregressive process $s(n)$ corrupted with white Gaussian noise $u(n)$. That is

$$x(n) = s(n) + u(n). \tag{12}$$

For the sake of explanation, we ignore the effects of truncation of the response of an all-pole system and write the z transform of $s(n)$ as

$$S(z) = \frac{GE(z)}{A(z)} \tag{13}$$

where $E(z)$ is the z transform of the excitation sequence $e(n)$ and $G/A(z)$ is the z transform of the all-pole system corresponding to the autoregressive process. Now

$$X(z) = S(z) + U(z) = \frac{V(z)}{A(z)} \tag{14}$$

where

$$V(z) = GE(z) + U(z)A(z). \tag{15}$$

The group delay function of $X(z)$ in terms of the group delay functions of $V(z)$ and $A(z)$ is given by

$$\tau_X(\omega) = \tau_V(\omega) - \tau_A(\omega). \tag{16}$$

The Fourier transform of $x(n)$ is given by

$$X(\omega) = \frac{GE(\omega) + A(\omega)U(\omega)}{A(\omega)}. \tag{17}$$

For low noise levels the first term $GE(\omega)$ dominates and hence the group delay function $\tau_X(\omega)$ of $X(z)$ behaves almost like the group delay function $\tau_S(\omega)$ of $S(z)$ (noise free case). For high noise levels two cases have to be considered separately: a) Regions (say $\bar{\boldsymbol{R}}$) of frequency where the values of $|A(\omega)|$ are not small (i.e., not near zero) and also the shape of $|A(\omega)|$ curve is smooth, and b) regions (say $\boldsymbol{R}$) of frequencies where the values of $|A(\omega)|$ are so small that the first term in $V(z)$, namely, $GE(z)$, dominates. In regions $\bar{\boldsymbol{R}}$ the group delay function $\tau_V(\omega)$ corresponding to the numerator polynomial of (14) behaves as it would for any noise sequence. That is, there will be large positive and negative spikes depending on the roots of $V(z)$ in the region $\bar{\boldsymbol{R}}$. In the regions $\boldsymbol{R}$ the group delay function $\tau_V(\omega)$ still will have large amplitude spikes of either polarity, but this time they are contributed by the roots of $V(z)$ in the region $\boldsymbol{R}$, where the first term in $V(z)$ dominates. Thus, in both the regions $\bar{\boldsymbol{R}}$ and $\boldsymbol{R}$ the group delay function behaves as it would for a noise sequence, but due to different sources. The most important point is that the spiky nature of the group delay function $\tau_X(\omega)$ is not affected significantly by the presence of $A(z)$ in the numerator. This is the reason why the first term $\tau_V(\omega)$ in $\tau_X(\omega)$ is distinct from the second term $\tau_A(\omega)$. The characteristics of the second term can still be estimated by suppressing the spikes in the overall group delay function $\tau_X(\omega)$. That this works even for very low noise levels is obvious from this argument.

The basis for the new spectrum estimation procedure is to suppress the large amplitude spikes in $\tau_X(\omega)$ due to $\tau_V(\omega)$ in order to highlight the desired components $\tau_A(\omega)$. To suppress the spikes due to noise, it is necessary to identify their locations and then reduce their amplitudes. The locations and amplitudes of these spikes are not known. We can take advantage of the behavior of the spectrum where the noise zeros contribute to sharp nulls. If we can derive a spectrum with nearly flat spectral envelope, then it contains mainly the spectral shape contributed by the zeros. A cepstrally smoothed spectrum can be used to obtain an estimate of the zero spectrum (a spectrum with flat envelope). The size of the cepstrum window is not very critical. We have chosen a cepstrum window size of 10 samples throughout our studies. The group delay function can be multiplied by the estimated zero spectrum (or spectrum of signal having approximately flat envelope) to suppress the noise spikes in $\tau_X(\omega)$. The resulting group delay function is an estimate $-\hat{\tau}_A(\omega)$ of the group delay spectral component corresponding to the desired autoregressive process. Assuming that $-\hat{\tau}_A(\omega)$ corresponds to a minimum phase all-pole system, the spectrum can be derived using the relation between the group delay function and the cepstral coefficients [5]. The resulting spectrum is the estimated spectrum from the Fourier transform phase generated by the proposed method.

IV. Illustrations

We consider two types of problems for illustration.

Example 1: Autoregressive process in noise (estimation of the AR spectrum)

$$x_1(n) = s(n) + u(n) \tag{18}$$

$$s(n) = -\sum_{k=1}^{4} a_k s(n-k) + Ge(n) \tag{19}$$

where the excitation $e(n)$ is a white Gaussian noise of variance unity and $u(n)$ is an additive noise with variance dependent upon the desired signal-to-noise ratio (SNR). The values of the coefficients are: $a_1 = -2.760$, $a_2 = 3.809$, $a_3 = -2.654$, and $a_4 = 0.924$.

Example 2: Two sinusoids in noise (estimation of frequencies of the sinusoids)

$$x_2(n) = \sqrt{10}\exp[j2\pi(0.10)n] + \sqrt{20}\exp[j2\pi(0.15)n] + u(n) \tag{20}$$

where $u(n)$ is additive white Gaussian noise with the variance dependent upon the SNR. These examples are similar to the ones used in [2] for discussion of periodogram estimates.

We assume a sampling frequency of 10 kHz and number of samples $N = 256$ for example 1, and $N = 100$ for example 2. Different realizations of $x_1(n)$ and $x_2(n)$ are obtained by using a different noise sequence each time. The group delay function $\tau_X(\omega)$ is computed using the following formula [11]:

$$\tau_X(\omega) = -\mathrm{Im}\frac{d(\log X(\omega))}{d\omega} = \frac{X_R(\omega)Y_R(\omega) + Y_I(\omega)X_I(\omega)}{|X(\omega)|^2} \tag{21}$$

1. Let $x(n)$ be the given M-pt causal sequence. Compute $y(n) = nx(n)$.
2. Compute the N-pt ($N >> M$) discrete Fourier transform (DFT) $X(k)$ and $Y(k)$ of the sequences $x(n)$ and $y(n)$ respectively, $k = 0,1,\ldots,N-1$.
3. Compute cepstrally smoothed spectrum $S(k)$ of $|X(k)|^2$.
4. Compute the zero spectrum $\hat{Z}(k)$ by dividing $|X(k)|^2$ by $S(k)$.
5. Compute the modified group delay function $\tau_o(k)$ as

$$\tau_o(k) = \frac{X_R(k)Y_R(k) + X_I(k)Y_I(k)}{|X(k)|^2}.\hat{Z}(k), k=0,1,\ldots,N-1.$$

where R and I denote the real and imaginary parts respectively.

(a)

1. Compute the estimate of the weighted cepstrum from $\tau_o(k)$ as follows. Compute N-pt IDFT of $\tau_o(k)$

$$\hat{c}(n) = \mathrm{IDFT}[\tau_o(k)]\ , n = 0,1,\ldots,N-1.$$

2. Form the sequence $c_1(n)$

$$c_1(0) = 0$$
$$\left.\begin{aligned} c_1(n) &= \hat{c}(n)/n \\ c_1(N-n+1) &= c_1(n) \end{aligned}\right\}, 1 \le n \le N/2.$$

3. Compute the N-pt DFT of $c_1(n)$

$$X_1(k) = \mathrm{DFT}[c_1(n)], \qquad k = 0,1,\ldots,N-1.$$

4. Compute

$$\ln|X_s(k)| = \mathrm{Real}[X_1(k)].$$

$2{*}\ln|X_s(k)|$ is the estimated smoothed spectrum as obtained from the modified group delay.

(b)

Fig. 4. (a) Algorithm for computing the modified group delay function. (b) Algorithm for computing the smoothed spectrum from the modified group delay function.

where the subscripts R and I denote the real and imaginary parts of the Fourier transform. $X(\omega)$ and $Y(\omega)$ are the Fourier transforms of the sequences $x(n)$ and $y(n) = nx(n)$, respectively. The procedure for computing the modified group delay function and the estimated spectrum for a given sequence of samples $x(n)$ is given in Fig 4.

Figs. 5–7 give the periodogram, group delay function and the new spectrum estimates of the autoregressive process from the noisy signal (SNR = 20 dB) of example 1. Figs. 5(a), 6(a), and 7(a) show the plots for a single realization of clean data. Figs. 5(b), 6(b), and 7(b) show the plots for 50 realizations of noisy data. Figs. 5(c), 6(c), and 7(c) show the averaged plots. It is to be noted, as expected, that the periodogram estimate has large variance (Fig. 5(b)). Reduction of variance by averaging several periodograms introduces large bias [2]. The variance is significantly reduced in the spectrum estimated by group delay method as can be seen from Figs. 5(b) and 7(b). The reduction in variance and bias due to the proposed method of spectrum estimation can be seen clearly from Figs. 7(b) and (c). The averaging reduces the dynamic range in periodogram (Figs. 5(a) and (c)) whereas averaging the estimated spectrum from group delay does not seem to significantly affect the dynamic range (Figs. 7(a) and (c)).

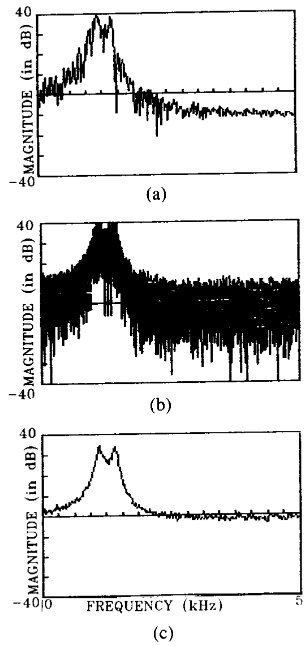


Fig. 5. Periodogram estimation of spectrum for an autoregressive process in noise (N = 256, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (c) Average of realizations.

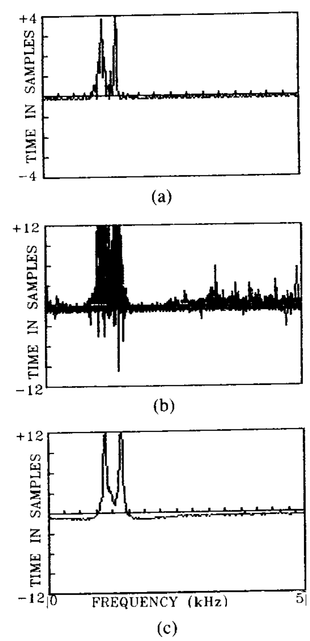


Fig. 6. Estimated group delay function for an autoregressive process in noise (N = 256, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (c) Average of realizations.

Although we have not discussed the theory, we have applied our method for estimating sinusoids in noise. The

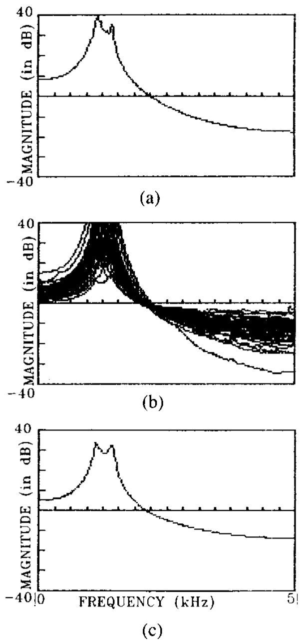


Fig. 7. Estimated spectrum from group delay function for an autoregressive process in noise (N = 256, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (c) Average of realizations.

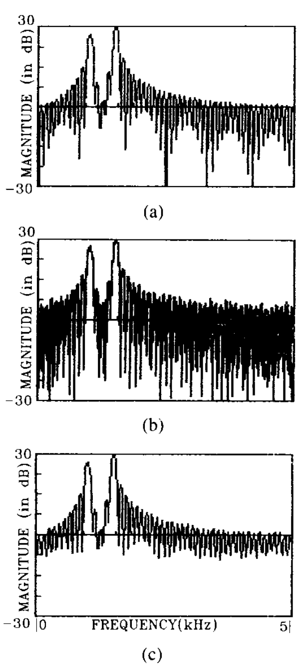


Fig. 8. Periodogram estimation of spectrum for sinusoids in noise (N = 100, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (c) Average of realizations.

results are shown in the plots given in Figs. 8–10 for SNR = 20 dB. The proposed method works well even for estimating sinusoids in the presence of noise. The same

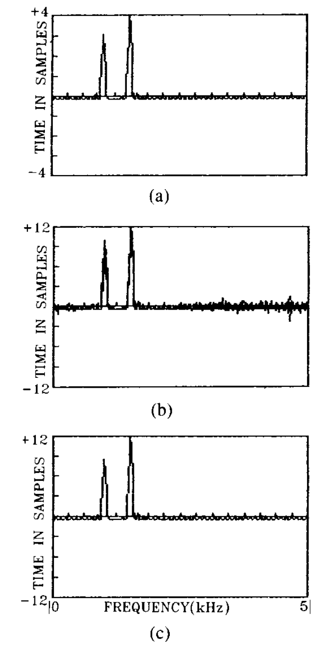


Fig. 9. Estimated group delay function for sinusoids in noise (N = 100, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (c) Average of realizations.

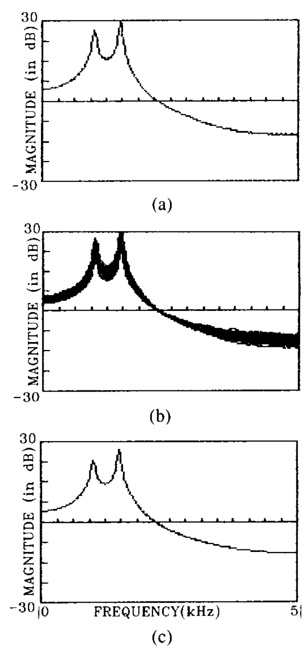


Fig. 10. Estimated spectrum from group delay function for sinusoids in noise (N = 100, SNR = 20 dB). (a) Single realization (clean signal). (b) Fifty overlaid realizations. (b) Average of realizations.

general conclusions as for the autoregressive process hold good for sinusoidal process.

Note that the finite data window also produces large spikes in the group delay function. But multiplication by the estimated zero spectrum suppresses the sidelobe ef-

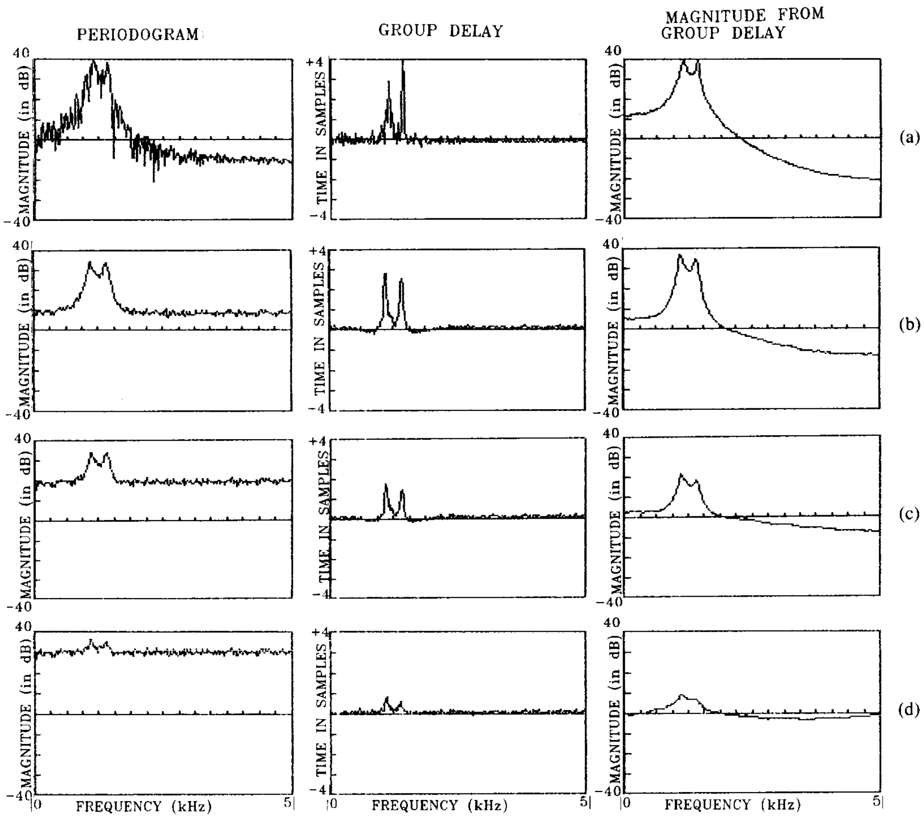


Fig. 11. Estimated spectra from group delay functions for different noise levels for an autoregressive process in noise (N = 256, the data is presented as an average of 50 realizations for noisy data). (a) Clean signal. (b) SNR = 10 dB. (c) SNR = 0 dB. (d) SNR = −10 dB.

fects of the window also. This way the estimated spectrum from the group delay function is less dependent on the type of window. However, the resolution of the spectral peaks is dependent on the size of the window. The effective window size is reflected in the width of the spikes in the group delay function, the width is smallest for the rectangular window and largest for the Nuttall window. The windows chosen for this study are the Hamming, Hanning, Nuttal, and rectangular windows. For a detailed discussion of these windows refer to [3].

Figs. 11 and 12 show the results of the estimated spectra for different noise levels (SNR = 10 dB, SNR = 0 dB, SNR = −10 dB). Figs. 11(b)–(d) show the averaged plots for 50 realizations (256 samples in each realization) of an AR process in noise. The plots show that our method restores significant features and the dynamic range, up to SNR = 0 dB, although there are some spurious peaks (though less significant) at higher noise levels. Figs. 12(b)–(d) show the averaged plots for 50 realizations (100 samples in each realization) of sinusoids in noise. The conclusions are similar to the case of the AR process in noise. These results show that the proposed method works even at high noise levels.

Note that model-based AR spectrum estimation will not work for noisy data [2]. Fig. 13 gives a comparison of the performance of our method of spectrum estimation with Burg's method [2]. The data consists of 256 samples of AR process in noise (single realization). Burg's method uses an eighth-order model. Note that the group delay function method preserves the resolution properties of the periodogram, with much less variance, even for low SNR. Unlike the periodogram method, the group delay method restores the dynamic range of the AR process even at high noise levels. Model-based techniques fail to resolve the peaks at high noise levels (SNR < 5 dB). If the order of the model is increased, more spurious peaks will be generated. It is interesting to note that even for a single realization, the dynamic range is almost restored and the fluctuation due to noise and data windows are almost absent in the estimated spectrum using the group delay method.

V. Conclusions

In summary, we have proposed a method of spectrum estimation that a) reduces fluctuations caused by the variance of noise and sidelobes due to window, b) has less effect on the bias since no averaging is involved to reduce the variance, c) restores the dynamic range and preserves the resolution of a periodogram estimate, d) works even

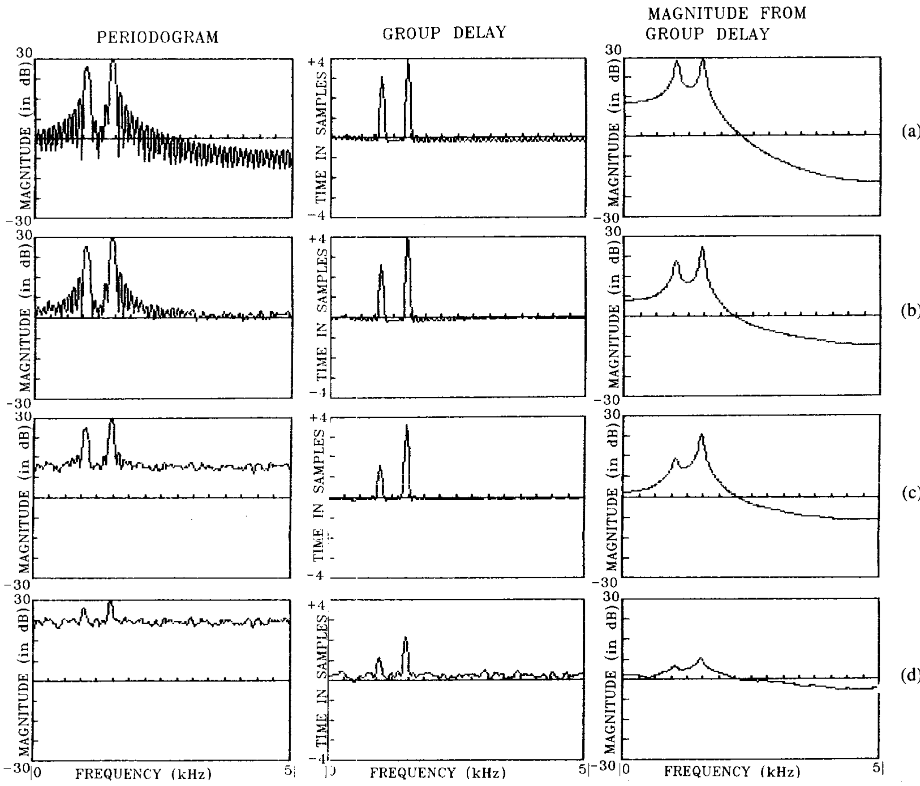


Fig. 12. Estimated spectra from group delay functions for different noise levels for sinusoids in noise (N = 100, the data is presented as an average of 50 realizations for noisy data). (a) Clean signal. (b) SNR = 10 dB. (c) SNR = 0 dB. (d) SNR = −10 dB.

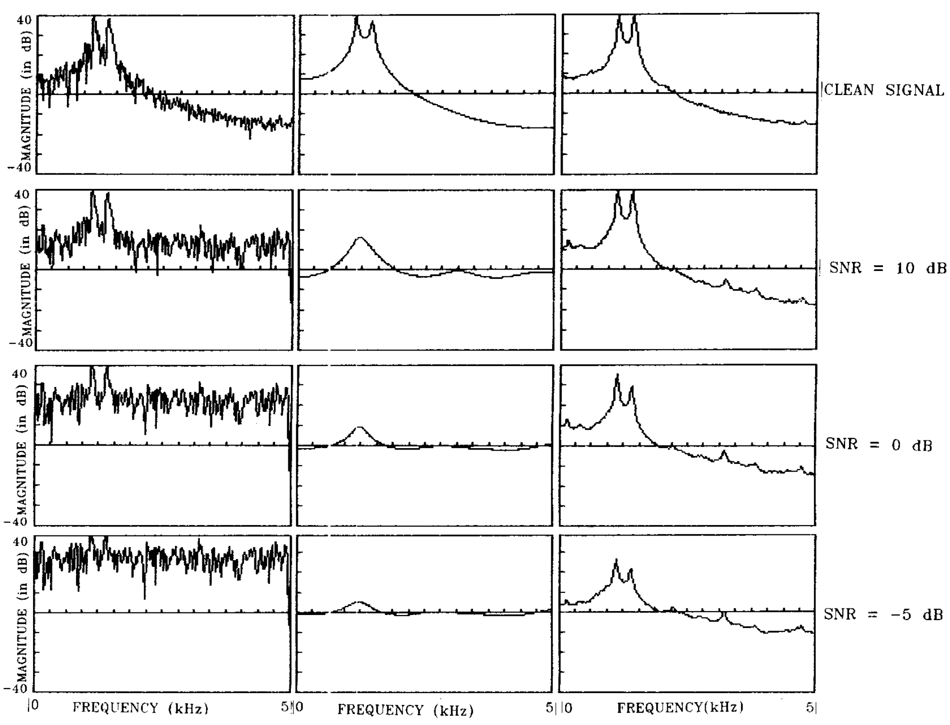


Fig. 13. Comparison of spectrum estimation by group delay method with some standard spectrum estimation methods for different noise levels. Data consists of 256 samples of autoregressive process in noise. (a) Periodogram. (b) Burg (order = 8). (c) Magnitude from group delay.

for high noise levels, e) performs better than model-based methods for noisy data, because resolution does not depend on factors like model order, and f) spurious peaks are nearly absent even at high noise levels. However, comparison with model-based methods for short data records is not apt, because knowledge of the model definitely gives a better resolution than the periodogram estimate. Thus, the proposed technique in its present form is not suitable for very short data record analysis.

We have given the theoretical basis only for the case of spectrum estimation using group delay function for an autoregressive process in noise, although we have demonstrated that the proposed method works equally well for sinusoids in noise. We have not addressed the problem of resolution of spectral peaks obtained by this method. We have also not derived expressions for bias and variance of the estimates. Only qualitative discussion through examples has been given.

In our opinion, for the first time an attempt has been made to process the FT phase for spectrum estimation. Studies in this paper show the potential of using FT phase for various applications [9], [10].

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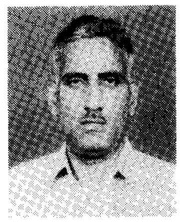

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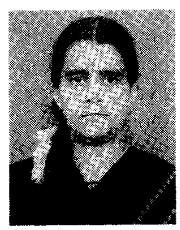

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